Math 8 honors: Training Problems on Prime Factorization, Number of Factors, Sum of all Factors

1. Given that $N = 2^4 \times 3^5 \times 5^6$, match each item on the left with the correct expression on the right:

i) The number of factors, ii) Sum of all the factors, iii) Number of ODD factors, iv) Number of Even Factors, vi) Sum of all Even Factors vii) Sum of all ODD Factors viii) Number of factors that are perfect squares ix) Sum of all factors that are perfect squares	a) $(5+1)\times(6+1)$ b) $4\times(5+1)\times(6+1)$ c) $(4+1)\times(5+1)\times(6+1)$ d) $(3)\times(3)\times(4)$ e) $\left(\frac{2^5-1}{2-1}\right)\times\left(\frac{3^6-1}{3-1}\right)\times\left(\frac{5^7-1}{5-1}\right)$ f) $\left(\frac{2^5-2}{2-1}\right)\left(\frac{3^6-1}{3-1}\right)\left(\frac{5^7-1}{5-1}\right)$ g) $\left(2^0+2^2+2^4\right)\left(3^0+3^2+3^4\right)\left(5^0+5^2+5^4+5^6\right)$ h) $\left(\frac{3^6-1}{3-1}\right)\left(\frac{5^6-1}{5-1}\right)$

2. Given each value of "N", find the following:
   i) The number of factors, ii) Sum of all the factors,
   iii) Number of ODD factors, iv) Number of Even Factors,
   vi) Sum of all Even Factors vii) Sum of all ODD Factors
   viii) Number of factors that are perfect squares ix) Sum of all factors that are perfect squares

a) $N = 2^4 \times 3^5 \times 5^6 \times 7^3$

b) $N = 2^2 \times 3^3 \times 5^4 \times 7^5$

c) $N = 12 \times 14 \times 15 \times 16$

d) $N = 10!$

e) $N = 2^3 \times 3^4 \times 4^5 \times 6^7$

f) $N = p_1{}^a \times p_2{}^b \times p_3{}^c$ [p1, p2, and p3 are prime numbers; a,b,c are positive integers]

3. How can you find the sum of the series? What is this series called? $1+2+4+8+16+32+...+1024$

4. How can you find the sum of the series? What is this series called? $1+3+9+27+81+243+729$?

5. How is the sum of a series used in finding the sum of all the factors of "N"? Explain?

6. When getting the number of factors of "N", why do we take each exponent of the prime factors, add one to each one, and then multiply it? Please explain:

7. Given that $N = 2^4 \times 3^5 \times 5^6 \times 7^3$. If one of the factors of "N" were chosen what is the probability that this factor is an even number? Explain:

   b) What is the probability that this factor selected will be an ODD number? Explain

Gauss 2002

A perfect number is an integer that is equal to the sum of all of its positive divisors, except itself. For example, 28 is a perfect number because $28 = 1 + 2 + 4 + 7 + 14$. Which of the following is a perfect number?

(A) 10 **(B)** 13 **(C)** 6 **(D)** 8 **(E)** 9

Gauss 2005

If $N = 2^5 \times 3^2 \times 7 \times \square$ and 100 divides evenly into N, which of the following numbers could be placed in the box?

(A) 5 **(B)** 20 **(C)** 75 **(D)** 36 **(E)** 120

Cayley 2018

If n is a positive integer, the symbol $n!$ (read "n factorial") represents the product of the integers from 1 to n. For example, $4! = (1)(2)(3)(4)$ or $4! = 24$. If x and y are integers and $\dfrac{30!}{36^x 25^y}$ is equal to an integer, what is the maximum possible value of $x + y$?

(A) 10 **(B)** 47 **(C)** 17 **(D)** 26 **(E)** 13

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6. How many ways are there to choose integers a, b and c with $a < b < c$ from the list 1, 5, 8, 21, 22, 27, 30, 33, 37, 39, 46, 50 so that the product abc is a multiple of 12?